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AN EXISTENCEOF HYPERONIN HYDROGEN-LIKE ATOM FOR $\Xi^- - Pb$ SYSTEM

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Abstract

The purpose of our research is to calculate the energy of hydrogen-like $\Xi^- - Pb$ atom for various states by solving the Schrödinger equation. We assumed as $\Xi^- - Pb$ atom is liked an ordinary hydrogen atom in which the electron is replaced by a negative xi hyperon. By assuming above approximation, we also calculated the energy of hydrogen-like Pb^{81+} ionized atom.And then, we determined the radius of hydrogen-like $\Xi^- - Pb$ atom to know about their structure. We found that the energy of hydrogen-like $\Xi^- - Pb$ atom is greater than Pb^{81+} ionized atom for various orbits and the inclusion of xi reduces the size of the atom in such a way the smaller radius is obtained. In addition, we observed that the radius of $\Xi^- - Pb$ atom is less than nuclear radius for lower orbits. It is found that the existence of atomic state and nuclear state cannot be distinguished in lower orbits. In this way, the existence of atomic and nuclear hybrid state can be found in lower orbits of $\Xi^- - Pb$ atom.

Keywords: energy of hydrogen-like atom, radius of hydrogen-like atom.

Introduction

The hydrogen atom is a simple mathematical problem in quantum mechanics, but any atom with more than one electron is difficult that an exact solution is impossible. For the two-electron atom, helium, very elaborate approximate solution have been set up by Hylleraas, which gives result agreeing with experimental error. This agreement has convinced physicists that Schrödinger's equation for the many-body problem provides the correct starting point for a study of more complicated atoms. But the method used for helium is too complicated to apply to atoms with more than two electrons and the approximations must be made for many electron atoms. A good starting point is provided by assuming that each electron moves in a central or spherically symmetrical, force field, produced by the nucleus and other electrons.

The Hydrogenic Core Model for Atoms with One-Valence Electron

The alkali atoms are approximately hydrogen like. They all have one valence electron. It will suffice to say that (Z-1) of the electrons are arranged in closed shells. So far as the electric field outside the closed shells is concerned, the nuclear charge +Ze, shielded by the (Z-1) electron carrying a charge of –(Z-1)e, is equivalent to +e, the same as the hydrogen atom. Thus the valence electron is essentially in a hydrogen-like field. However, inside the closed shells the shielding effect of (Z-1) electrons becomes less and less as we approach the centre and eventually becomes zero in the region just outside the nucleus. For large r the valance electron has an electrostatic potential $-\frac{e^2}{r}$, this expresses the fact that the nuclear charge +Ze is screened by the core of (Z-1) electrons. For small r, potential approaches $-\frac{Ze^2}{r}$ corresponding to an unscreened nucleus. Overall, the electrostatic attraction towards the nucleus is always greater than for hydrogen.

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Hydrogen-Like Atom

The spectra of all atoms or ions with only one electron should be the same expect for the factor Z^2 and the Rydberg number. The spectrum of hydrogen should thus explain those of the ions He^{+}, Li^{2+}, Be^{3+} or any other ions which have only one electron. For Li^{2+}, Be^{3+} and still heavier highly ionized atoms, spectral lines have been observed which can be calculated by multiplying the frequencies of the lines of the H atom by Z^2 and insertion of the corresponding Rydberg constant. In 1916, the collected spectroscopic experience concerning the hydrogen-similarity of these spectra was generalized in displacement theorem of Sommerfeld and Kossel, which states: the spectrum of any atom is very similar to the spectrum of the singly charged positive ion which follows it in the periodic table.

Hydrogen-like heavy atoms are the heavy atoms from which all the electrons expect one have been removed. Hydrogen-like heavy atoms can be prepared by accelerating the singly-ionized atoms to high energies and passing them through a thin foil; their electrons are "stripped off" on passing through the foil. For example, in order to strip all the electrons from a uranium atom and produce U^{92+} ions, they must be accelerated to energies greater than 10 GeV. By permitting the U^{92+} ions to recapture one electron each, one can then obtain hydrogen-like ion U^{91+} .The corresponding spectral lines are emitted as the captured electron makes transitions from orbits of high n to lower orbits. In present work, we assumed above approximation, a $\Xi^{-}-Pb$ atom is like an ordinary hydrogen atom in which the electron is replaced by a negative xi. We calculated the energy of hydrogen-like Pb^{81+} ionized atom and $\Xi^{-}-Pb$ atom forvarious states by solving the Schrodinger equation. And then, we also calculated the radii of $\Xi^{-}-Pb$ atom to know about their structure.

Solving the Schrödinger Equation for Hydrogen Atom

The Schrödinger equation for the hydrogen atom is

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi \tag{1}$$

$$\left[\nabla^2 + \frac{2\mu}{\hbar^2}(E - V)\right]\psi = 0 \tag{2}$$

Where the potential energy V of the electron in the hydrogen atom is

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} \tag{3}$$

$$\nabla^2 = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] \tag{4}$$

Substitution equation (4) into equation (2)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{2\mu}{\hbar^2}[E - V(r)]\psi = 0 \tag{5}$$

The wave function ψ is variable-seperable, i.e, ψ can be written as

$$\psi(r,\theta,\varphi) = P(\theta)Q(\varphi)R(r) \tag{6}$$

where $P(\theta)$ is a function of θ only, $Q(\varphi)$)of φ only and $R(r)$ of r only.

Substitution equation (6) into equation (5)

$$\frac{P(\theta)Q(\varphi)}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + \frac{R(r)Q(\varphi)}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}P(\theta)\right) + \frac{P(\theta)R(r)}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Q(\varphi)$$

$$+\frac{2\mu}{\hbar^2}[E - V(r)]P(\theta)Q(\varphi)R(r) = 0 \tag{7}$$

Dividing by $P(\theta)Q(\varphi)R(r)$ on both sides,

$$\frac{1}{R(r)r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + \frac{1}{P(\theta)r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}P(\theta)\right) + \frac{2\mu}{\hbar^2}[E - V(r)]$$

$$= -\frac{1}{Q(\varphi)r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Q(\varphi) \tag{8}$$

Multiplying equation (8) by $r^2\sin^2\theta$, we get

$$\frac{\sin^2\theta}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{\sin\theta}{P(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) + \frac{2\mu\, r^2\sin^2\theta}{\hbar^2}[E - V(r)]$$

$$= -\frac{1}{Q(\varphi)}\frac{d^2}{d\varphi^2}Q(\varphi) \tag{9}$$

According to equation (9), the R. H. S and L. H. S should equal to some arbitrary constant and we use m_ℓ^2.

Therefore R. H. S of equation (9) becomes

$$-\frac{1}{Q(\varphi)}\frac{d^2}{d\varphi^2}Q(\varphi) = m_\ell^2 \tag{10}$$

Similarly, L. H. S of equation (9) becomes

$$\frac{\sin^2\theta}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{\sin\theta}{P(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) + \frac{2\mu r^2\sin^2\theta}{\hbar^2}[E - V(r)] = m_\ell^2 \tag{11}$$

Dividing equation (11) by $\sin^2\theta$, we get

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{2\mu r^2}{\hbar^2}[E - V(r)] = \frac{m_\ell^2}{\sin^2\theta} - \frac{1}{P(\theta)\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) \tag{12}$$

Similarly, L. H. S and R. H. S should equal to some arbitrary constant.

Therefore, the R. H. S of equation (12) becomes

$$\frac{m_\ell^2}{\sin^2\theta} - \frac{1}{P(\theta)\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) = L^2 \tag{13}$$

and L. H. S becomes

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\frac{2\mu r^2}{\hbar^2}[E-V(r)]=L^2 \tag{14}$$

Substitution the potential term

$$V(r)=-\frac{Ze^2}{4\pi\varepsilon_0 r} \tag{15}$$

We obtain the equation

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\frac{2\mu r^2}{\hbar^2}\left[\frac{Ze^2}{4\pi\varepsilon_0 r}+E\right]=L^2 \tag{16}$$

By multiplying with $\frac{R(r)}{r^2}$, equation (16) becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\left[\frac{2\mu}{\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0 r}+E\right)-\frac{\ell(\ell+1)}{r^2}\right]R(r)=0$$

Since the reduce mass $\mu=m$, multiplying with $\frac{\hbar^2}{2m}$, we get

$$\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left(\frac{Ze^2}{r}+E\right)R-\frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}R=0 \tag{17}$$

where $4\pi\varepsilon_0=1$ in atomic unit.

After solving mathematical steps,we obtain the energy of electron for the n^{th} orbit.

$$E_n=-\frac{Z^2me^4}{2\hbar^2}\frac{1}{n^2} \tag{18}$$

where, E_n = the total energy of the electron

m = mass of electron

$\hbar=\frac{h}{2\pi}$ (h=Planck constant)

e = electron charge

n = 1, 2, 3…

We calculated the energy levels of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom by using the above equation.

Calculation of the Radius for Hydrogen Atom

The Coulomb force between a stationary nucleus with charge +Ze and an orbiting electron with charge –e is

$$F=-\frac{kZe^2}{r^2} \tag{19}$$

From Newton's second law of motion, the centripetal force

$$F = -\frac{mv^2}{r} \quad (20)$$

From equation (19) and equation (20)

$$mv^2 = \frac{kZe^2}{r} \quad (21)$$

But, from Bohr's postulate

$$mvr = n\hbar$$

$$v^2 = \frac{n^2\hbar^2}{m^2r^2} \quad (22)$$

Substituting the equation (22) into equation (21) we getthe radius of electron for the n^{th} orbit.

$$r_n = \frac{n^2\hbar^2}{mkZe^2} \quad (23)$$

Where, $r_n =$ the radius of the circular orbit

m = mass of electron

v = velocity of electron

k = constant = $9\times10^9\,Nm^2C^{-2}$

$\hbar = \frac{h}{2\pi}$ (h=Planck constant)

e = electron charge

n = 1, 2, 3 ….

By using the above equation, we calculated the radius of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom for various orbits.

Results and Discussions

The Energy of Hydrogen-Like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom

Hydrogen-like heavy atoms are the heavy atoms from which all the electrons except one have been removed. We assumed as $\Xi^- - Pb$ atom is like an ordinary hydrogen atom in which the electron is replaced by a negative xi hyperon. And then we calculated the energy of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom for various states by solving the Schrödinger equation. From these results, we observed that the energy of $\Xi^- - Pb$ atom is greater than Pb^{81+} ionized atom for each orbits. The higher the orbit, the greater the energy for these atoms. The calculated energy for these atoms is given in Table (1) and the energy levels of Pb^{81+} ionized atom as shown in Figure (1). Then the energy levels $\Xi^- - Pb$ atom as shown in Figure (2).

The Radius of Hydrogen-Like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom

And then we also calculated the radius of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom as given in Table (2). In our calculation, the radius of $\Xi^- - Pb$ atom is very smaller than Pb^{81+} ionized atom for each orbit. The higher the orbits, the greater the size of these atoms. The inclusion of xi reduces the size of the atom in such a way the small radius is obtained. Moreover, we found that the radius of $\Xi^- - Pb$ atom is less than the nuclear radius for lower orbit. Therefore, the existence of atomic state and nuclear state are mixture. The mixture of the atomic state and nuclear state is called hybrid state. So we cannot distinguish the existence of the atomic state or the nuclear state of hydrogen-like $\Xi^- - Pb$ atom for lower orbits.

Table 1 Comparison between energy of Hydrogen–like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom for various orbits

Orbital quantum number	Energy (MeV)	
	Hydrogen-like Pb^{81+} ionized atom	Hydrogen-like $\Xi^- - Pb$ atom
1	- 0.0911624	- 236.4217125
2	- 0.0227906	- 59.1054281
3	- 0.0101292	- 26.2690792
4	- 0.0056976	- 14.7763570
5	- 0.0036465	- 9.4568685
6	- 0.0025323	- 6.6572698

Table 2 Comparison between radii of Hydrogen–like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom for various orbits

Orbital quantum number	Radius (fm)	
	Hydrogen – like Pb^{81+} ionized atom	Hydrogen – like $\Xi^- - Pb$ atom
1	647. 6339	0.2497
2	2590.5356	0.9988
3	5828.7051	2.2473
4	10362.1424	3.9952
5	16190.8475	6.2425
6	23314.8204	8.9892

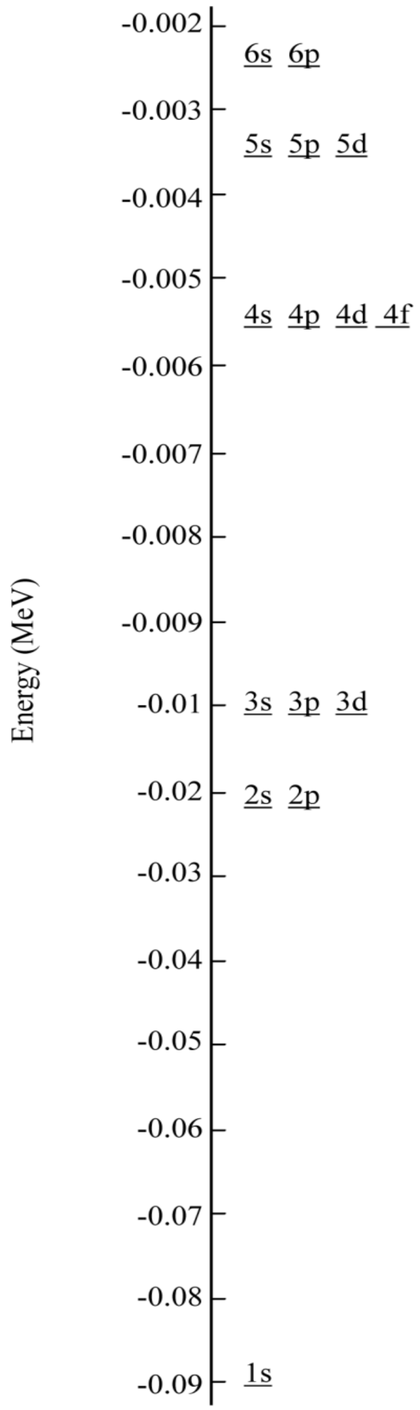

Figure 1 Energy levels of the Pb^{81+} ionized atom

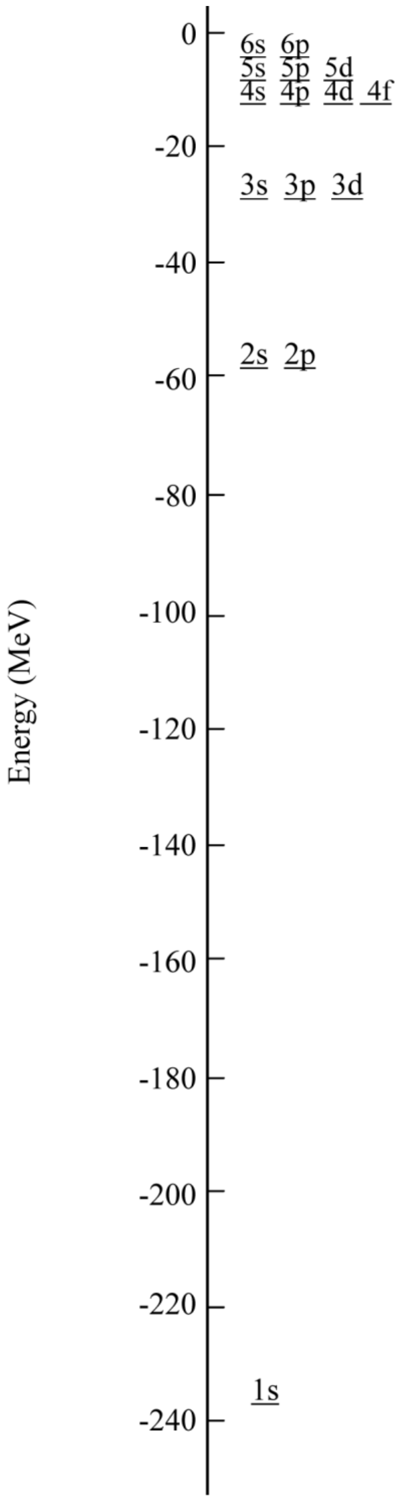

Figure 2 Energy levels of the $\Xi^{-} - \mathrm{Pb}$ atom atom

We calculated the energy of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom for various states by solving the Schrödinger equation. And then, we also calculated the radii of hydrogen-like Pb^{81+} ionized atom and $\Xi^- - Pb$ atom to know about their structure. In our calculation, we assumed as $\Xi^- - Pb$ atom is liked an ordinary hydrogen atom in which the electron is replaced by a negative xi hyperon. From the calculations, we concluded that the radius of $\Xi^- - Pb$ atom is less than nuclear radius and we cannot distinguish the existence of atomic state and nuclear state (hybrid state).

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